Analysis of the implementation of the Leslie matrix on the female population growth rate model

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ABSTRACT

Demographic problems in Indonesia are important because Indonesia is ranked fourth as the country with the largest population. One way to determine future population growth is to predict female population growth. The Leslie matrix is a model used to predict and determine female population growth. The general form of the Leslie matrix is a square matrix in which the first row entries are the female fertility rates (a_i), the subdiagonal entries are the female survival rates (b_i), and the remaining entries are zero. The purpose of the study was to predict the rate of female population growth in Indonesia. The results showed that the Leslie matrix, influenced by the initial population, female fertility rate, and female survival rate, had a dominant eigenvalue of 1.001, indicating that the rate of female population growth in Indonesia tends to increase.

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1. INTRODUCTION

Demography, or population, is a field of study that examines human population dynamics [1]. Population is a problem related to the structure, number, age, gender, marriage, pregnancy, birth, death, distribution, mobility, religion, and the strength and resilience of political, economic, social, and cultural institutions [2]. Based on research [2] related to the problems faced in the population sector, namely the problem of population growth. Population growth is characterized by changes in the number of people [3]. Population growth is the change in the number of people in a given area over a given period of time. Population growth is also known as the population growth rate. Population growth rate indicators are very helpful for projecting a region's future population [4]. Population is one of the problems in a state and government environment, including in the Indonesian Republic, because population data is essential [5]. In this regard, population-related problems in Indonesia are significant, given that Indonesia is the fourth-largest country in the world [6]. In addition, in the context of national development, one aspect to consider is population [4].

Population growth is influenced by various things such as births, deaths, and migration. In general, population growth is a continuous process. However, it is also necessary to study population growth from a discrete time. The use of discrete patterns is also based on population observations, which are usually carried out at specified time intervals, such as a day, a week, or a period adjusted to the researcher's research plan. Based on these considerations, the development of population growth models should also be evaluated using discrete solutions [7]. One way to predict future population growth is to forecast the growth of the female population, since women can reproduce. Women influence the increase in birth rates [8]. As the number of people increases, the population will also increase. The growth of women in a region is also important, as the 2005-2025 RPJM development plan aims to improve the quality of human resources, including women's roles in development [9]. The female population can be characterized by modeling its growth using the Leslie matrix [10]. The Leslie matrix has a unique pattern: a square matrix with the elements in the first row corresponding to the fertility rate, the subdiagonal elements corresponding to the survival rate, and the remaining elements equal to zero [11].

The Leslie matrix is used to predict population growth [12]. The Leslie Matrix was discovered by the ecologist P.H. Leslie in 1945. Leslie stated that, for simplicity, it was assumed that the same age range within a given time period consisted only of women who were the object of the Leslie Matrix calculation [9]. Therefore, the Leslie matrix is also referred to as the population growth model [13]. The Leslie Matrix was discovered by an ecologist named P.H. Leslie in 1945. Factors that affect population growth are female fertility rates, female survival rates, and women's life spans from population classes [14]. The population used in the calculation of the Leslie matrix model is the female population. In the Leslie matrix, it is assumed that there is no migration from this population, that the survival rate is not zero, and that at least one age class has a positive fertility rate [11]. Objectives: The purpose of the study is to determine the process of forming the Leslie matrix and the growth rate of the female population in Indonesia using the Leslie matrix model assisted by the Maple 18 application.

2. METHOD

The research used a quantitative research design. The research procedure for analyzing the implementation of the Leslie matrix in the female population growth rate model in Indonesia is as follows [15].

1. Female population data collection

In this study, data on the female population, female fertility rates, and the survival rates of Indonesian women are needed [13]. Data on the number of women in Indonesia was obtained from the National BPS from 2018 to 2020. The collected data are the number of females grouped by gender and age interval.

2. Formation of the Leslie matrix

The first step in forming the Leslie matrix is to determine the vector of the initial age distribution. The initial distribution vector is the vector of the female population in the 0th year in each age group. In this study, the initial distribution vector was a female population for each age group. The population can be represented as a column vector [11]. After determining the initial distribution vector, the next step is to specify the assumptions for the Leslie matrix. The assumptions used to form the Leslie matrix are fertility rates and women's survival rates. In determining the level of female fertility, the average number of girls born to each woman in the next age group is used, and in determining the level of female survival, the average number of girls born to each woman in the next age group is used. Based on the assumptions used, the Leslie matrix is formed with the first entry equal to the female fertility rate, the main subdiagonal entry equal to the female survival rate, and all other entries equal to zero [11]. The Leslie matrix model is used to predict future population sizes and growth rates of the female population. The growth model with the Leslie matrix used the formula.

$$
\begin{gathered}
\mathbf{x}^{(\mathbf{1})} = L\mathbf{x}^{(\mathbf{0})} \\
\mathbf{x}^{(\mathbf{2})} = L\mathbf{x}^{(\mathbf{1})} = L^2\mathbf{x}^{(\mathbf{0})} \\
\mathbf{x}^{(\mathbf{3})} = L\mathbf{x}^{(\mathbf{2})} = L^3\mathbf{x}^{(\mathbf{0})} \\
\vdots \\
\mathbf{x}^{(\mathbf{k})} = L\mathbf{x}^{(\mathbf{k-1})} = L^k\mathbf{x}^{(\mathbf{0})}
\end{gathered}
$$

In the Leslie matrix model, it is assumed that the population growth rate is only caused by fertility rates (a_i), Survival Rate (b_i) women, and it is considered that there is no migration in or out of the population [14]. The general form of the Leslie matrix model is as follows.

$$
L = \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ b_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & b_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_{n-1} & 0 \end{bmatrix}
$$

With:

a_i : Fertility rate or the average number of girls born to each woman when the mother is in the 1^{st} age class where $a_i \geq 0$ for $i = 1, 2, 3, \dots$

b_i : Female survival rate in the age group i with $i = 1, 2, 3, \dots, n-1$

The eigenvalues of the Leslie matrix are used to determine long-term population dynamics and changes in growth rates [3]. Although the Leslie matrix can describe the number of female populations at different times in the future, the equation does not directly provide an overview of the rate of population growth. Therefore, the eigenvalues of the Leslie matrix are required. The eigenvalues of the matrix are the roots of the characteristic polynomials. To provide an overview of the growth process, the eigenvalues of the Leslie matrix are used. The eigenvalue of L is the root of its characteristic polynomial [16]. Polynomials characteristic of the Leslie matrix are

$$
p(\lambda) = |\lambda I - L|
$$

$$
p(\lambda) = \left| \lambda \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \dots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ b_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & b_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_{n-1} & 0 \end{bmatrix} \right|
$$

$$p(\lambda) = \left\| \begin{bmatrix} \lambda & 0 & 0 & 0 & \dots & 0 \\ 0 & \lambda & 0 & 0 & \dots & 0 \\ 0 & 0 & \lambda & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \dots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ b_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & b_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_{n-1} & 0 \end{bmatrix} \right\|$$

$$p(\lambda) = \left\| \begin{bmatrix} \lambda - a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ b_1 & \lambda - 0 & 0 & \dots & 0 & 0 \\ 0 & b_2 & \lambda - 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & b_{n-1} & 0 \end{bmatrix} \right\|$$

$$p(\lambda) = \lambda^n - a_1\lambda^{n-1} - a_2b_1\lambda^{n-2} - a_3b_1b_2\lambda^{n-3} - \dots - a_nb_1b_2 \dots b_{n-1} \quad (1)$$

Using the above function, the equation of its characteristics $p(\lambda) = 0$, so that

$$p(\lambda) = 0$$
$$\lambda^n - a_1\lambda^{n-1} + a_2b_1\lambda^{n-2} + a_3b_1b_2\lambda^{n-3} - \dots - a_nb_1b_2 \dots b_{n-1} = 0$$
$$\lambda^n = a_1\lambda^{n-1} + a_2b_1\lambda^{n-2} + a_3b_1b_2\lambda^{n-3} - \dots + a_nb_1b_2 \dots b_{n-1}$$
$$\frac{\lambda^n}{\lambda^n} = \frac{a_1\lambda^{n-1}}{\lambda^n} + \frac{a_2b_1\lambda^{n-2}}{\lambda^n} + \frac{a_3b_1b_2\lambda^{n-3}}{\lambda^n} - \dots + \frac{a_nb_1b_2 \dots b_{n-1}}{\lambda^n}$$
$$1 = \frac{a_1}{\lambda^n} + \frac{a_2b_1}{\lambda^n} + \frac{a_3b_1b_2}{\lambda^n} + \dots + \frac{a_nb_1b_2 \dots b_{n-1}}{\lambda^n}$$

To analyze the roots of these polynomial equations, it will be easier to recognize the following functions.

$$q(\lambda) = \frac{a_1}{\lambda} + \frac{a_2b_1}{\lambda^2} + \frac{a_3b_1b_2}{\lambda^3} + \dots + \frac{a_nb_1b_2 \dots b_{n-1}}{\lambda^n} \quad (2)$$

Using this function, the characteristic equation $p(\lambda) = 0$ can be written with $q(\lambda) = 1$ for $\lambda \neq 0$.

All of the a_i and b_i is a positive value, $q(\lambda)$ monotonous reduction to λ greater than zero [11]. $q(\lambda)$ has a vertical asymptote on the $\lambda = 0$ and close to zero when $\lambda \to \infty$. It can be concluded that the eigenvalues of the Leslie matrix are distinct, with a single positive value. There is a λ unique, for example, $\lambda = \lambda_1$, in such a way that $q(\lambda_1) = 1$. There are three cases in the dominant eigenvalue of the Leslie matrix [11], namely

A. If $\lambda_1 > 1$ then the growth rate of a population increases
B. If $\lambda_1 = 1$ Therefore, the growth rate of a population is relatively constant.
C. If $\lambda_1 < 1$ then the growth rate of a population decreases

3. Analysis of the implementation of the Leslie matrix on the female growth rate model

The third stage in implementing the Leslie matrix in the growth rate model is to calculate its dominant eigenvalue. In this case, the Maple 18 application will compute the dominant eigenvalue of the Leslie matrix. The next stage is to analyze the dominant eigenvalue produced. Suppose the dominant eigenvalue of the Leslie matrix is positive. In that case, the growth rate of women tends to increase, whereas if the dominant eigenvalue is negative, the growth rate tends to decrease.

3. RESULTS AND DISCUSSION

In the Leslie matrix model, it is assumed that the population growth rate is only caused by fertility rates (a_i), Survival Rate (b_i) women, and it is considered that there is no migration in or out of the population. In this study, the data used included data on the number of the female population in Indonesia in 2018 and 2020, which were grouped by age to determine the initial age of the population, birth data in the form of ASFR to determine the fertility rate of women, and mortality data for the female population to increase women's survival. The data used are shown in **Table 1**.

Table 1. Total Female Population in 2018 and 2020

Age Class	*Age Interval (Years)*	*Total Population in 2018 (Thousands)*	*Total Population in 2020 (thousands)*
1	0-4	11.622,5	10.778,8
2	5-9	11.679,4	10.799,0
3	10-14	11.146,6	10.746,1
4	15-19	10.864,2	10.816,9
5	20-24	10.726,2	11.050,1
6	25-29	10.494,6	10.945,2
7	30-34	10.258,0	10.795,5
8	35-39	10.164,0	10.354,3
9	40-44	9.551,3	9.928,5
10	45-49	8.657,3	8.996,9
11	50-54	7.536,5	7.874,0
12	55-59	6.205,7	6.574,5
13	60-64	4.663,9	5.117,8
14	65-69	3.224,4	3.772,6
15	70-74	2.279,7	2.374,9
16	75+	2.804,9	2.617,0

The age grouping of women in the table is adjusted to data obtained from the BPS (Central Statistics Agency). The class grouping is divided into 16 classes. The fertility rate and the resilience level of the female population influence the Leslie matrix model. In **Table 2,** data on the formation of the Leslie matrix are obtained.

Table 2. Value of Fertility Rate and Survival Rate of Women

Age Interval (Years)	*Fertility Rate* (a_i)	*Resilience Level* (b_i)
0-4	0	0,997
5-9	0	0,997
10-14	0	0,998
15-19	0,1	0,997
20-24	0,21	0,997
25-29	0,3	0,996
30-34	0,22	0,995
35-39	0,13	0,992
40-44	0,05	0,987
45-49	0,01	0,98
50-54	0	0,969
55-59	0	0,954
60-64	0	0,929

65-69	*0*	*0,889*
70-74	*0*	*0,821*
75+	*0*	*0,532*

Based on the values of the fertility rate and female survival rate, a Leslie matrix model is obtained for the female population growth rate model, namely a $16\ \times 16$ square matrix. The leslie matrix obtained is a matrix 16×16 consisting of the first line element, namely the fertility level (a_i) and the main subdioagonal is the survival rate (b_i) female population. The matrix model obtained is

$$L =$$
$$\begin{bmatrix}
0 & 0 & 0 & 0{,}1 & 0{,}21 & 0{,}3 & 0{,}22 & 0{,}13 & 0.05 & 0{,}01 & 0 & 0 & 0 & 0 & 0 & 0 \\
0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0{,}998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}996 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0{,}995 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}992 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}969 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}954 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}929 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}889 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}821 & 0
\end{bmatrix}$$

From above Matriks Leslie, a model of female population growth in Indonesia was obtained in the following year with the equation

$$\mathbf{x}^{(\mathbf{1})} = L\mathbf{x}^{(\mathbf{0})}$$
$$\mathbf{x}^{(\mathbf{2})} = L\mathbf{x}^{(\mathbf{1})} = L^2\mathbf{x}^{(\mathbf{0})}$$
$$\mathbf{x}^{(\mathbf{3})} = L\mathbf{x}^{(\mathbf{2})} = L^3\mathbf{x}^{(\mathbf{0})}$$
$$\vdots$$
$$\mathbf{x}^{(\mathbf{k})} = L\mathbf{x}^{(\mathbf{k-1})} = L^k\mathbf{x}^{(\mathbf{0})}$$

L	:	Leslie Matrix Model
$\mathbf{x}^{(\mathbf{k})}$	:	Total number of female population in year-k
$\mathbf{x}^{(\mathbf{0})}$	:	Initial number of female population

Based on the table, it is obtained

$$\mathbf{x}^{(0)} = \begin{bmatrix} 10788,8 \\ 10799 \\ 10746,1 \\ 10816,9 \\ 11050,1 \\ 10945,2 \\ 10795,5 \\ 10354,3 \\ 9928,5 \\ 8996,9 \\ 7874 \\ 6574,5 \\ 5117,8 \\ 3772,6 \\ 2374,9 \\ 2617 \end{bmatrix}$$

Here are some calculations to determine the number of female population for the next two years, which are as follows:

$$\mathbf{x}^{(1)} = L.\mathbf{x}^{(0)}$$

$$\begin{bmatrix}
0 & 0 & 0 & 0,1 & 0,21 & 0,3 & 0,22 & 0,13 & 0.05 & 0,01 & 0 & 0 & 0 & 0 & 0 & 0 \\
0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0,996 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0,995 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,992 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,98 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,969 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,954 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,929 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,889 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,821 & 0
\end{bmatrix}
\begin{bmatrix} 10788,8 \\ 10799 \\ 10746,1 \\ 10816,9 \\ 11050,1 \\ 10945,2 \\ 10795,5 \\ 10354,3 \\ 9928,5 \\ 8996,9 \\ 7874 \\ 6574,5 \\ 5117,8 \\ 3772,6 \\ 2374,9 \\ 2617 \end{bmatrix}
=
\begin{bmatrix} 10993,2 \\ 10746,5 \\ 10766,6 \\ 10455,1 \\ 10748,5 \\ 11016,9 \\ 10901,4 \\ 10741,5 \\ 10271,5 \\ 9799,4 \\ 8816,9 \\ 7629,9 \\ 6272,1 \\ 4754,4 \\ 3353,8 \\ 1949,8 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{x}^{(1)} &= 10993,2 + 10746,5 + 10766,5 + 10455,1 + 10748,5 + 11016,9 + 10901,4 + \\
&\quad 10741,5 + 10271,5 + 9799,4 + 8816,9 + 7629,9 + 6272,1 + 4754,4 + 3353,8 \\
&\quad +1949,8 \\
&= 139256,5
\end{aligned}$$

$$\mathbf{x}^{(2)} = L^2.\mathbf{x}^{(0)}$$

$$\begin{bmatrix}
0 & 0 & 0 & 0,1 & 0,21 & 0,3 & 0,22 & 0,13 & 0.05 & 0,01 & 0 & 0 & 0 & 0 & 0 & 0 \\
0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0,997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0,996 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0,995 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,992 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,98 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,969 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,954 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,929 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,889 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,821 & 0
\end{bmatrix}^2
\begin{bmatrix} 10788,8 \\ 10799 \\ 10746,1 \\ 10816,9 \\ 11050,1 \\ 10945,2 \\ 10795,5 \\ 10354,3 \\ 9928,5 \\ 8996,9 \\ 7874 \\ 6574,5 \\ 5117,8 \\ 3772,6 \\ 2374,9 \\ 2617 \end{bmatrix}
=
\begin{bmatrix} 11021,6 \\ 10960,3 \\ 10714,2 \\ 10745,1 \\ 10423,8 \\ 10752,1 \\ 10972,9 \\ 10846,9 \\ 10655,6 \\ 10137,9 \\ 9603,4 \\ 8543,6 \\ 7278,9 \\ 5826,8 \\ 4226,7 \\ 2753,5 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{x}^{(2)} &= 11021,6 + 10960,3 + 10714,2 + 10745,1 + 10423,8 + 10752,1 + 10972,9 + \\
&\quad 10846,9 + 10665,6 + 10137,9 + 9603,4 + 8543,6 + 7278,9 + 5826,7 \\
&\quad +2753,5 \\
&= 145473,3
\end{aligned}$$

$$\mathbf{x}^{(3)} = L^3.\mathbf{x}^{(0)}$$

$$\mathbf{x}^{(3)} =$$

$$\begin{bmatrix}
0 & 0 & 0 & 0{,}1 & 0{,}21 & 0{,}3 & 0{,}22 & 0{,}13 & 0.05 & 0{,}01 & 0 & 0 & 0 & 0 & 0 & 0 \\
0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0{,}998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0{,}997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}996 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0{,}995 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}992 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}987 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}969 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}954 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}929 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}889 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}821 & 0
\end{bmatrix}^3
\begin{bmatrix} 10788{,}8 \\ 10799 \\ 10746{,}1 \\ 10816{,}9 \\ 11050{,}1 \\ 10945{,}2 \\ 10795{,}5 \\ 10354{,}3 \\ 9928{,}5 \\ 8996{,}9 \\ 7874 \\ 6574{,}5 \\ 5117{,}8 \\ 3772{,}6 \\ 2374{,}9 \\ 2617 \end{bmatrix}
=
\begin{bmatrix} 10947{,}4 \\ 10988{,}5 \\ 10927{,}4 \\ 10692{,}8 \\ 10721{,}8 \\ 10392{,}5 \\ 10709{,}1 \\ 10918 \\ 10760{,}1 \\ 10517{,}1 \\ 9935{,}2 \\ 9305{,}7 \\ 8150{,}6 \\ 6762{,}1 \\ 5179{,}9 \\ 3470{,}1 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{x}^{(3)} &= 10947{,}4 + 10998{,}5 + 10927{,}3 + 10692{,}8 + 10712{,}8 + 10392{,}5 \\
&\quad +10709{,}1 + 10918 + 10760{,}1 + 10517{,}1 + 9935{,}2 + 9305{,}7 \\
&\quad +8150{,}6 + 6762{,}1 + 5180 + 3470{,}1 \\
&= 150379{,}3
\end{aligned}$$

The Leslie matrix calculation results show that the female population for the next two years, 2022, will be 139,256.5 thousand; the female population in the second year, 2024, will be 145,473.3 thousand; and the female population in 2026 will be 150,379.3 thousand. The female population increases annually.

The next step will be calculating the dominant eigenvalue of the Leslie matrix. This dominant eigen value will determine the value of the growth rate in Indonesia. The growth rate of the female population will increase, decrease, or remain relatively constant. The formula of the dominant eigenvalue of the Leslie matrix is

$$p(\lambda) = |\lambda I - L|$$

By using the Maple 18 application, the eigenvalue of the Leslie matrix is obtained, which is as follows:

$$= \begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\
1.00061880746849 \\
0.478514543879181 + 0.708406633352675I \\
0.478514543879181 - 0.708406633352675I \\
-0.000162398576332767 + 0.572436134401630I \\
-0.000162398576332767 - 0.572436134401630I \\
-0.309650200684948 + 0.514224118848680I \\
-0.309650200684948 - 0.514224118848680I \\
-0.366597453461634 + 0.220977217760259I \\
-0.366597453461634 - 0.220977217760259I \\
-0.0604827789781023
\end{bmatrix}$$

The eigenvalues obtained are real and complex. The magnitude of a real number is the absolute value of the number itself. In contrast, the magnitude of a complex number is obtained by $|z| =$

$\sqrt{x^2 + y^2}$, for a complex number $z = x + yi$ where x, y are real numbers. This the positive dominant eigenvalues of the Leslie matrix are obtained, namely $\lambda_1 = 1.00061880746849 \approx 1{,}001$. eigen value $\lambda_1 = 1{,}001 < 1$, This means that the growth rate of the female population in Indonesia tends to increase.

Based on the results of the Leslie matrix calculation for the growth of the Female population, the implementation of the Leslie matrix in the analysis of population growth shows that this model is effective in predicting population dynamics based on age structure, birth rate, and survival rates for each age group. Population growth has increased, as evidenced by the main eigenvalue being greater than one, indicating a long-term growth trend. The Leslie matrix also shows the age group that contributes most to the increase in population from this calculation, namely the 30-34 age group, which can serve as a basis for formulating demographic policies, such as education, health, and employment planning. Assuming that the birth rate and survival probability remain constant, this model projects exponential population growth. However, it should be noted that in real applications, social, economic, and government policy changes can affect the parameters in the Leslie matrix. Therefore, regular data updates and parameter adjustments are essential to maintain prediction accuracy.

4. CONCLUSION

The application of the Leslie matrix to population growth analysis shows that this model is effective at predicting population dynamics based on age structure, birth rates, and survival rates for each age group. The growth rate of the female population in Indonesia using the Leslie matrix model was obtained from the dominant eigenvalue of the Leslie matrix, namely $\lambda_1 = 1.00061880746849 \approx 1{,}001$. eigen value $\lambda_1 = 1{,}001 < 1$, This means that the growth rate of the female population in Indonesia tends to increase. The Leslie matrix also shows the age group that contributes most to the population increase from this calculation: the 30-34 age group. This can serve as a basis for formulating demographic policies, such as education, health, and employment planning.

When applying the Leslie matrix to predict the female population, it is assumed that only three factors are used: population age range, growth rate, and female survival rate. However, population migration can also influence population changes in a region. The author hopes that future research will modify the Leslie matrix to include population migration.

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